

Modelling the time variation of the surface differential rotation in AB Doradus and LQ Hydrae

A. F. Lanza*

INAF-Osservatorio Astrofisico di Catania, Via S. Sofia, 78, 95123 Catania, Italy

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Sequences of Doppler images of the young, rapidly rotating late-type stars AB Dor and LQ Hya show that their equatorial angular velocity and the amplitude of their surface differential rotation vary versus time. Such variations can be modelled to obtain information on the intensity of the azimuthal magnetic stresses within stellar convection zones. We introduce a simple model in the framework of the mean-field theory and discuss briefly the results of its application to those solar-like stars.

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1 Introduction

Doppler imaging techniques allow us to measure the surface differential rotation in rapidly rotating late-type stars by tracking the longitudinal motion of starspots located at different latitudes (Collier Cameron 2007). Specifically, the surface angular velocity Ω at colatitude θ is assumed to be given by a solar-like law:

$$\Omega(\theta) = \Omega_{\text{eq}} - d\Omega \cos^2 \theta, \quad (1)$$

where Ω_{eq} is the equatorial angular velocity and $d\Omega$ is the pole-equator angular velocity difference. Ω_{eq} and $d\Omega$ can be measured by fitting the shear of starspots along sequences of photospheric images covering successive rotations. Alternatively, Ω and $d\Omega$ can be included as free parameters in a code that reproduces the line profile distortions due to starspots, thus obtaining their values by a suitable χ^2 minimization when a sufficiently long time series of line profiles is available.

The long-term monitoring of surface differential rotation of the two late-type dwarfs AB Doradus and LQ Hydrae shows that their equatorial angular velocity and surface shear are functions of the time (see Donati et al. 2003a; Jeffers et al. 2007). It is interesting to note that the variations of Ω_{eq} and $d\Omega$ are compatible with an internal angular velocity uniform on cylindrical surfaces co-axial with the rotation axis (Donati et al. 2003a).

2 Modelling differential rotation variations

Differential rotation is the result of the re-distribution of angular momentum in a stellar convection zone under the action of meridional circulation, Reynolds stresses (i.e., the

correlations of the components of the turbulent velocity in a rotating star), and torque by the Lorentz force (quantified by the azimuthal Maxwell stresses). The shear associated with differential rotation is opposed by the turbulent viscosity present in the convection zone that tends to make the star rotate rigidly.

In order to model the time variation of the differential rotation, let us consider the equation for the angular momentum conservation in an inertial reference frame in the framework of the mean-field theory (e.g., Rüdiger & Hollerbach 2004):

$$\frac{\partial}{\partial t}(\rho r^2 \sin^2 \theta \Omega) + \nabla \cdot \Theta = 0, \quad (2)$$

where r is the distance from the centre of the star, t the time, ρ the density, $\Omega(r, \theta, t)$ the angular velocity and Θ the angular momentum flux vector given by:

$$\Theta = (\rho r^2 \sin^2 \theta \Omega) \mathbf{u}_{(m)} + r \sin \theta \langle \rho \mathbf{u}' u'_{\varphi} \rangle - \frac{r \sin \theta}{\tilde{\mu}} (\mathbf{B} B_{\varphi} + \langle \mathbf{B}' B'_{\varphi} \rangle), \quad (3)$$

where $\mathbf{u}_{(m)}$ is the meridional circulation, \mathbf{u}' the fluctuating velocity, \mathbf{B} the mean magnetic field, \mathbf{B}' the fluctuating magnetic field and $\tilde{\mu}$ the magnetic permeability; angular brackets $\langle \rangle$ indicate the Reynolds mean, defining the mean quantities. Note that in Eq. (3) the first term specifies the angular momentum transport by meridional circulation, the second by Reynolds stresses and the third by Maxwell stresses, respectively.

The Reynolds stresses can be written as:

$$\langle \rho u'_i u'_j \rangle = -\eta_t \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \Lambda_{ij}, \quad (4)$$

where the first term represents the effect of the turbulent viscosity and the second one is the so-called Λ -effect due to the correlation of the fluctuating velocity components produced

* Corresponding author: e-mail: nuccio.lanza@oact.inaf.it

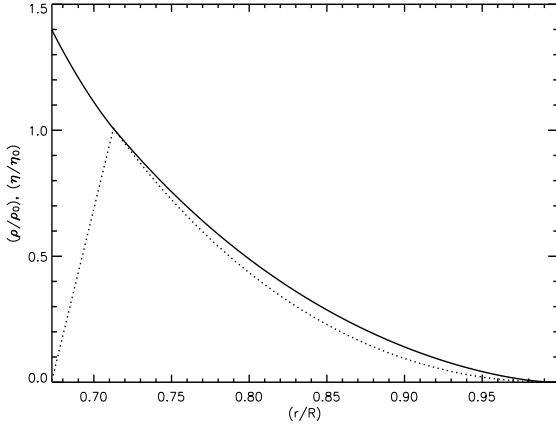


Fig. 1 The dynamic turbulent viscosity, computed according to standard mixing-length theory (dotted line) and the density (solid line), normalized to their respective values at the base of the convection zone, in the model for a solar-like star adopted by Lanza (2007). The convection zone extends from $0.713 R$ to the surface, whereas an overshoot layer is assumed to extend between 0.673 and $0.713 R$ with a linearly increasing η_t .

by the action of the Coriolis force in a rotating star (Rüdiger 1989).

The conservation of the total angular momentum of the convection zone leads to the following boundary conditions:

$$\Theta_r = 0 \text{ for } r = r_c, R, \quad (5)$$

where r_c is the radius at the lower boundary of the convection zone and R is the radius of the star. We separate the contribution of the turbulent viscosity stress tensor from the other terms in Eq. (3) and adopt a dynamic turbulent viscosity that depends only on the radial co-ordinate, i.e., $\eta_t = \eta_t(r)$ (see Fig. 1 for details). In such a way, the equation for the angular velocity derived from Eqs. (2), (3) and (4) becomes:

$$\begin{aligned} \frac{\partial \Omega}{\partial t} - \frac{1}{\rho r^4} \frac{\partial}{\partial r} \left(r^4 \eta_t \frac{\partial \Omega}{\partial r} \right) \\ - \frac{\eta_t}{\rho r^2} \frac{1}{(1 - \mu^2)} \frac{\partial}{\partial \mu} \left[(1 - \mu^2)^2 \frac{\partial \Omega}{\partial \mu} \right] = S, \end{aligned} \quad (6)$$

where $\mu \equiv \cos \theta$ and the source term S is given by:

$$S = - \frac{\nabla \cdot \boldsymbol{\tau}}{\rho r^2 (1 - \mu^2)}, \quad (7)$$

$\boldsymbol{\tau}$ being a vector whose components are:

$$\begin{aligned} \tau_i = \rho r^2 \sin^2 \theta \Omega u_{(m)i} \\ + r \sin \theta \left[\Lambda_{i\varphi} - \frac{1}{\tilde{\mu}} (B_i B_\varphi + \langle B'_i B'_\varphi \rangle) \right]. \end{aligned} \quad (8)$$

The boundary conditions given by Eq. (5) can be recast as:

$$\frac{\partial \Omega}{\partial r} = 0 \text{ for } r = r_c, R. \quad (9)$$

The solution of the angular velocity equation (6) with the boundary conditions (9) can be expressed in terms of an appropriate Green function (see Lanza 2007 for details):

$$\begin{aligned} \Omega(r, \theta, t) &= \int_V [\nabla \cdot \boldsymbol{\tau}(r', \theta', t)] G(r, \theta, r', \theta') dV' = \\ &= \int_V (\boldsymbol{\tau} \cdot \nabla' G) dV', \end{aligned} \quad (10)$$

where the integration with respect to the variables r' and θ' is extended to the volume of the convection zone V and $\nabla' G$ is the gradient of G with respect to the variables r' and θ' . The second equality in Eq. (10) follows by the application of Gauss's Theorem and the vanishing of τ_r at the boundaries of the convection zone. Note that Eq. (10) is valid when the timescale of variation of $\boldsymbol{\tau}$ is much longer than the diffusion timescale of the convection zone $t_{\text{diff}} \simeq (R - r_c)^2 / \nu_t$, with $\nu_t \propto \eta_t / \rho$ the average kinematic turbulent viscosity of the convection zone. If this assumption is not valid, the solution can still be expressed as a linear combination of terms each depending on an appropriate Green function (see Lanza 2007). For the sake of simplicity, however, we prefer to work with Eq. (10), so we retain this assumption.

Note that the Green function G depends on η_t and ρ , but does not depend on the angular momentum transport terms.

We shall obtain information on the averaged intensity of the angular momentum transport vector $\boldsymbol{\tau}$ in the framework of two different hypotheses, i.e.: a) the internal angular velocity is uniform on cylindrical surfaces co-axial with the rotation axis (Taylor-Proudman regime); or, b) the torque is localized between, say, $r_1 \leq r \leq r_2$ within the convection zone.

3 Applications and results

Let us consider first the case of a stellar convection zone rotating in the Taylor-Proudman regime. This is probably the case for rapid rotators, like AB Dor and LQ Hya. In such a case, Eq. (10) can be written in the form:

$$\Delta \Omega(s, t) = \int_0^R \mathcal{C}(s') \tau_s(s', t) \frac{\partial G(s, s')}{\partial s'} ds', \quad (11)$$

where $\Delta \Omega$ is the variation of the angular velocity with respect to a reference rotation state (e.g., that with $\boldsymbol{\tau} = 0$), s is the distance from the stellar rotation axis in a cylindrical polar reference frame and $\mathcal{C}(s)$ the lateral area of the cylindrical surface of radius s within the convection zone. If we pose:

$$M = \max \left\{ \left| \frac{\partial G}{\partial s'} \right| \right\} \text{ in the convection zone,} \quad (12)$$

then, Eq. (11) can be applied to obtain the inequality (see Lanza 2006a for details):

$$\int_V |\tau_s| dV' \geq \frac{|\Delta \Omega|}{M}. \quad (13)$$

Star	$d\Omega$	$\min \{ B_s B_\varphi \}$	B_{\min}	B_{eq}	P_{diss}/L
AB Dor	59.6	0.0385	0.196	0.475	0.23
AB Dor	96.7	0.0624	0.250	0.475	0.60
LQ Hya	-48.7	0.0336	0.183	0.495	0.15
LQ Hya	200.9	0.1386	0.372	0.495	2.52

Table 1 $d\Omega$: surface shear in mrad d^{-1} ; B_{\min} : minimum magnetic field, obtained for $B_s = B_\varphi$; B_{eq} : mean equipartition field. Magnetic field intensities are given in Teslas.

Equation (13) provides us with a lower limit for the average radial angular momentum transport term in the convection zone. Under the additional hypotheses that the reference state corresponds to solid-body rotation with the total angular momentum of the star, and $\tau = -(s/\tilde{\mu})B_\varphi \mathbf{B}$, i.e., only the azimuthal mean-field Maxwell stresses contribute to the deviation of the differential rotation from the reference state (cf. Covas, Moss & Tavakol 2005), we find:

$$\min \{|B_s B_\varphi|\} = 8.22 \times 10^{-4} \left(\frac{\alpha_{\text{ML}}}{1.75} \right)^{4/3} \times \left(\frac{M}{M_\odot} \right)^{2/3} \left(\frac{L}{L_\odot} \right)^{1/3} \left(\frac{R}{R_\odot} \right)^{-5/3} |d\Omega| \text{ T}^2, \quad (14)$$

and the ratio of the dissipated power to the stellar luminosity (Lanza 2006a):

$$\frac{P_{\text{diss}}}{L} = 4.40 \times 10^{-5} \left(\frac{\alpha_{\text{ML}}}{1.75} \right)^{4/3} \left(\frac{M}{M_\odot} \right)^{2/3} \times \left(\frac{L}{L_\odot} \right)^{-2/3} \left(\frac{R}{R_\odot} \right)^{4/3} (d\Omega)^2; \quad (15)$$

where the surface shear $d\Omega$ is measured in mrad d^{-1} , α_{ML} is the ratio of the mixing-length to the pressure scale-height, M the mass, R the radius and L the luminosity of the star. The results of the application of these equations to AB Dor and LQ Hya are listed in Table 1, for $\alpha_{\text{ML}} = 1.75$ and adopted stellar parameters $M = 1.0 M_\odot$, $R = 1.05 R_\odot$, $L = 0.618 L_\odot$ for AB Dor and $M = 0.95 M_\odot$, $R = 0.95 R_\odot$ and $L = 0.506 L_\odot$ for LQ Hya, respectively. The first column in Table 1 indicates the name of the star, the second the observed surface shear, as obtained from Zeeman Doppler Imaging based on Stokes V data (see Donati et al. 2003a), the third the minimum Maxwell stress, the fourth the minimum mean field obtained for $B_s = B_\varphi$, the fifth the mean equipartition field derived from the average kinetic energy of convective motions computed according to the mixing-length theory, and the last the ratio of the dissipated power to the stellar luminosity. The minimum field strengths turn out to be smaller than the average equipartition values. However, Zeeman Doppler Imaging provides us with an estimate of the radial mean field $B_s \approx 0.01 \text{ T}$ in AB Dor and LQ Hya (Donati et al. 2003b), implying that the toroidal mean field is significantly stronger than the minimum values listed in Table 1, i.e., $B_\varphi \approx 3 - 14 \text{ T}$.

Such strong toroidal fields produce a radial Lorentz force that opposes the centrifugal force in a rotating star leading to a perturbation of its oblateness and therefore of the quadrupole moment of its outer gravitational potential. If the active star is a member of a close binary system, this leads to a variation of the orbital period through a mechanism originally proposed by Matese & Whitmire (1983) and further discussed by Lanza (2005) and references therein.

Note that the power dissipated in the turbulent convection zone of LQ Hya exceeds the stellar luminosity in the case of the largest observed shear, i.e., $d\Omega = 200.9 \text{ mrad d}^{-1}$. This does not pose a serious problem if the duration of such episodes of large shear is short in comparison to the phases with nearly rigid rotation because the large energy reservoir represented by the thermal content of the stellar convection zone will average out the energy loss over a timescale comparable with the Kelvin-Helmholtz timescale of the envelope. Moreover, the uncertainty in the determination of the largest $d\Omega$ excursion of LQ Hya (cf. Donati et al. 2003a) makes the estimate of the corresponding energy dissipation rate largely uncertain.

Let us now consider the case in which magnetic stresses are localized close to the base of the convection zone, without assuming a specific internal rotation regime. In this case, an expression similar to Eq. (13) can be obtained (cf. Lanza 2007). Specifically, we assume that $\tau = -(r \sin \theta / \tilde{\mu}) \mathbf{B} B_\varphi$ is localized in the overshoot layer, i.e., $0.67 \leq (r/R) \leq 0.71$ and that the differential rotation observed at the surface is equal to that at $r = 0.99R$.

In the case of the largest shear observed in LQ Hya, this yields a minimum average Maxwell stress over the overshoot layer $\{B_r B_\varphi\}_{\min} \simeq 0.063 \text{ T}^2$, which gives fields of the same order of magnitude of those obtained above under the Taylor-Proudman hypothesis.

4 Discussion and conclusions

We modelled the observed time variation of the differential rotation in AB Dor and LQ Hya under the hypotheses that the azimuthal Maxwell stresses rule the changes of their surface shear and the internal angular velocity depends only on the distance from the rotation axis (Taylor-Proudman regime). We obtained that the average intensity of the mean field Maxwell stresses is $|B_s B_\varphi| \sim 0.03 - 0.14 \text{ T}^2$, implying azimuthal mean fields $B_\varphi \sim (3 - 10) \text{ T}$ for $B_s \sim 0.01 \text{ T}$. Similar Maxwell stresses are obtained if the magnetic torque is assumed to be confined within the overshoot layer $0.67 \leq (r/R) \leq 0.71$ and no restrictions are imposed on the internal rotation law.

It is interesting to note that azimuthal magnetic fields of $3 - 10 \text{ T}$, occupying a sizeable fraction of the convection zone, have been invoked to explain orbital period changes observed in late-type active binaries (Lanza, Rodonò & Rosner 1998; Lanza & Rodonò 2004; Lanza 2005, 2006b). An α -effect related to an instability of the magnetic field itself (e.g., magnetic buoyancy instability, Bran-

denburg & Schmitt 1998; or magneto-rotational instability, Rüdiger et al. 2007) seems to be necessary to produce such super-equipartition fields, possibly acting in combination with differential rotation. The energy dissipated by turbulence, estimated according to standard mixing-length arguments, may exceed stellar luminosity in the case of the largest surface shear observed in LQ Hya. However, the thermal equilibrium of the convection zone can be significantly affected only if those large shear episodes last more than ~ 10 – 20 per cent of the time. Note also that a mixing-length estimate for the turbulent viscosity may not be appropriate for a rapidly rotating star (see Lanza 2006a for details).

In the present work, we adopted a point of view analogous to that of Covas et al. (2005) who modelled the variation of stellar differential rotation considering only the torque exerted by the Lorentz force and neglecting the roles of meridional flow and Λ -effect. As a matter of fact, it is difficult to evaluate the perturbations of the meridional flow and of the Reynolds stresses produced by the magnetic field because they depend critically on the approximations made in the treatment of stellar turbulence in a rotating star. Nevertheless, alternative models for the variation of differential rotation have been investigated, such as those based on a time-dependent component of the meridional flow (Rempel 2006, 2007) or the magnetic quenching of the Λ -effect (Rüdiger et al. 1986). The present approach can be generalized to obtain amplitudes of the perturbations of the corresponding terms in Eq. (8), but we shall not pursue this application here (see Lanza 2007).

Finally, it is interesting to note that some inference on the amplitude of variation of the surface shear in the case of very active stars can be obtained not only by means of Doppler imaging techniques, but also by an appropriate analysis of their long-term wide-band photometry (e.g., Rodonò et al. 2001; Messina & Guinan 2003). For example, in the case of LQ Hya, it is worth comparing the Doppler imaging results by Donati et al. (2003a) with the photometrically determined seasonal rotation periods by Kovári et al. (2004).

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